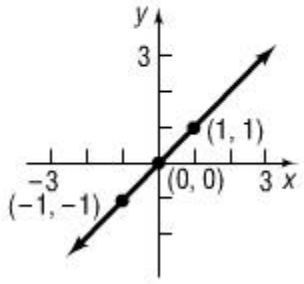
3.4 Library of Functions; Piecewise-Defined Functions

Identity Function

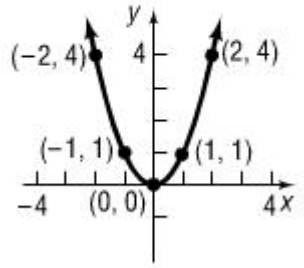
$f(x) = x$



- Domain: $\mathbb{R}$
- Range: $\mathbb{R}$
- Line with slope of $m = 1$
- Intercept: $(0,0)$
- Odd Function
- Increasing on $(-\infty,\infty)$
- Bisects Quadrants I and III
- Key Points: $(-2,-2),\ (-1,-1),\ (0,0),\ (1,1),\ (2,2)$

Square Function

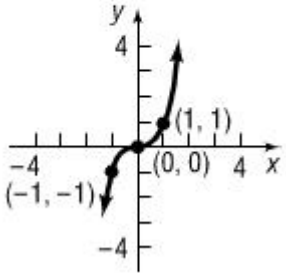
$f(x) = x^2$



- Domain: $\mathbb{R}$
- Range: $[0,\infty)$
- Parabola
- Intercept: $(0,0)$
- Even Function
- Decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$
- Key Points: $(-2,4),\ (-1,1),\ (0,0),\ (1,1),\ (2,4)$

Cube Function

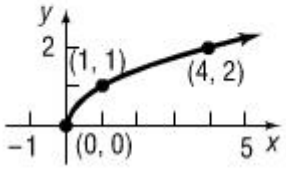
$f(x) = x^3$



- Domain: $\mathbb{R}$
- Range: $\mathbb{R}$
- Intercept: $(0,0)$
- Odd Function
- Increasing on $(-\infty,\infty)$
- Key Points: $(-2,-8),\ (-1,-1),\ (0,0),\ (1,1),\ (2,8)$

Square Root Function

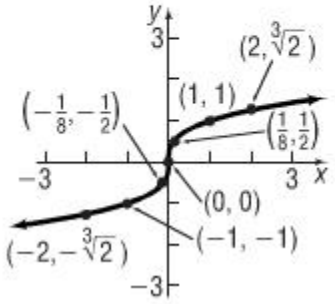
$f(x) = \sqrt{x}$



- Domain: $[0,\infty)$
- Range: $[0,\infty)$
- Intercept: $(0,0)$
- Neither even nor odd
- Increasing on $[0,\infty)$
- Minimum value of 0 at $x = 0$
- Key Points: $(0,0),\ (1,1),\ (4,2),\ (9,3)$

Cube Root Function

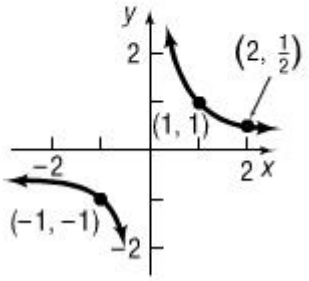
$f(x) = \sqrt[3]{x}$



- Domain: $\mathbb{R}$
- Range: $\mathbb{R}$
- Intercept: $(0,0)$
- Odd Function
- Increasing on $(-\infty,\infty)$
- Key Points: $(-8,-2),\ (-1,-1),\ (0,0),\ (1,1),\ (8,2)$

Reciprocal Function

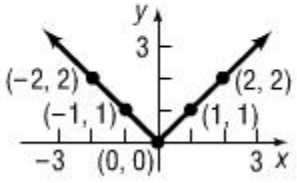
$f(x) = \dfrac{1}{x}$



- Domain: $(-\infty,0)\cup(0,\infty)$
- Range: $(-\infty,0)\cup(0,\infty)$
- No Intercepts
- Odd Function
- Decreasing on $(-\infty,0)$ and $(0,\infty)$
- Key Points: $(-2,-\frac{1}{2}),\ (-1,-1),\ (-\frac{1}{2},-2),\ (\frac{1}{2},2),\ (1,1),\ (2,\frac{1}{2})$

Absolute Value Function

$f(x) = |x|$



- Domain: $\mathbb{R}$
- Range: $[0,\infty)$
- Intercept: $(0,0)$
- Even Function
- Decreasing on $(-\infty,0)$ and increasing on $(0,\infty)$
- Minimum value of 0 at $x = 0$
- Key Points: $(-2,2),\ (-1,1),\ (0,0),\ (1,1),\ (2,2)$

Graphing Piecewise-Defined Functions

Sometimes a function is defined differently on different parts of its domain. When functions are defined by more than one equation, they are called ***piecewise-defined functions*.**

Examples: For the following functions:

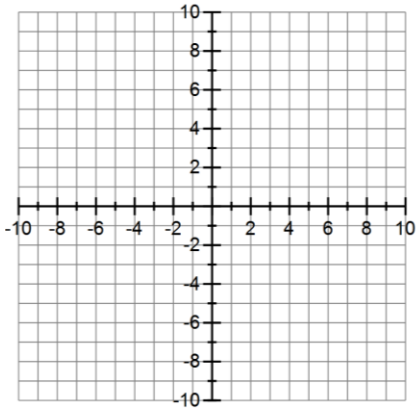
a) Graph the function.

b) Find the domain and range of the function.

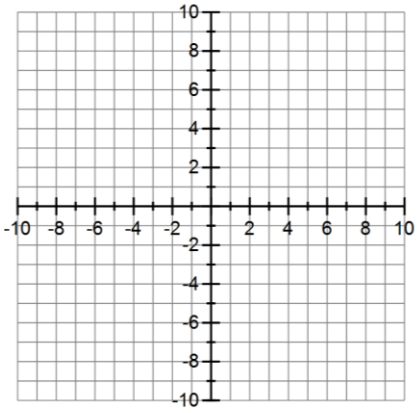
c) Locate any intercepts.

d) State whether the function is continuous on its domain.

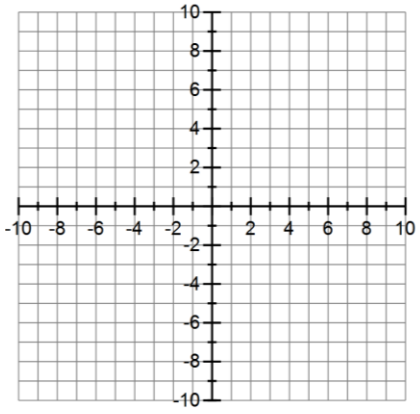
1) $f(x)=\begin{cases} x+3 & \text{if } x\le -1 \\ 2x & \text{if } x>1 \end{cases}$



2) $f(x)=\begin{cases} 2 & \text{if } -4<x<0 \\ x^2+2 & \text{if } x\ge 0 \end{cases}$



3) $f(x)=\begin{cases} 3-x & \text{if } -5\le x<-2 \\ \sqrt{x} & \text{if } 0<x<4 \\ 2x-6 & \text{if } x\ge 4 \end{cases}$



4) $f(x)=\begin{cases} |x| & \text{if } x<2 \\ 5 & \text{if } x=2 \\ -\frac{1}{2}x & \text{if } x>2 \end{cases}$